mental form:

(68) $$\mathrm{d}\sigma_i = \lambda\,\mathrm{d}\theta + 2\mu\,\mathrm{d}\varepsilon_i\,.$$

The definitions of S_i, E_i, p and θ lead to a restatement of Hooke's law as two equations:

(69) $$\mathrm{d}\overline{p} = -K\,\mathrm{d}\theta\,,$$

(70) $$\mathrm{d}S_i = 2\mu\,\mathrm{d}E_i\,.$$

In this form we separate those stresses which produce deformation from those which merely alter density so that the two relations can be discussed independently. It is commonly assumed that when nonelastic behavior occurs, it will appear in the deviator relation, not in hydrostatic compression. A notable exception is the porous solid, but that is not considered here. Following common practice we can write the constitutive relations for an elastic-plastic solid as:

(71) $$\mathrm{d}E_i = \mathrm{d}E_i^e + \mathrm{d}E_i^p\,,$$

(72) $$\mathrm{d}S_. = 2\mu\,\mathrm{d}E_i^e\,,$$

(73) $$\mathrm{d}\overline{p} = -K\,\mathrm{d}\theta\,,$$

where E_i^e and E_i^p are the elastic and plastic components of the strain deviator, respectively.

For a viscoelastic-plastic material, eqs. (71) and (73) apply as before, but (72) is replaced by

(74) $$\mathrm{d}S_i = 2\mu\,\mathrm{d}E_i^e + 2\eta\,\mathrm{d}\dot{E}_i^p\,,$$

where the dot indicates convective derivative with respect to time.

For a stress-relaxing solid we make the assumption that the plastic strain increment, in response to a change in stress, does not take its final value immediately. Its change is inhibited by a relaxation mechanism, undefined at this point. This process is represented by a relation of the form

(75) $$\mathrm{d}E_i^p/\mathrm{d}t = F_i(S_i,\,\varrho)/2\,.$$

The right-hand side of eq. (75) depends upon the amount by which E_i^p differs from its equilibrium value. Combining eqs. (71), (72) and (75) we arrive at the relation

(76) $$\mathrm{d}E_i/\mathrm{d}t - (1/2\mu)\,\mathrm{d}S_i/\mathrm{d}t = F_i(S_i\,,\,\overline{p})/2\,.$$

Equations (76) and (73) comprise a set of constitutive relations for a stress-relaxing material. Note particularly that the stress deviator, S_i, is entirely supported by the elastic strain, eq. (72). This distinguishes it fundamentally from the viscoelastic solid, eq. (74). Equations (71)-(73) for the elastic-plastic solid and eqs. (71), (73), and (76) for the elastic-plastic relaxing solid must be supplemented by a yield condition, *e.g.* the von Mises condition, eq. (41*a*). In uniaxial strain the yield condition can be incorporated in $F(S_i, \overline{p})$ in eq. (76). For this geometry eq. (76) can be replaced by a single equation:

$$\mathrm{d}p_x/\mathrm{d}t = a^2(\mathrm{d}\varrho/\mathrm{d}t) - F(p_x, \varrho)\,, \tag{77}$$

where a is the elastic sound speed at density ϱ.

In Sect. 2 we combined the flow equations, eqs. (1)-(3), under the assumption $p = p(\varrho)$, to form a set of characteristic equations, eqs. (25) and (26). A similar procedure can be executed in the present case. Combining eqs. (1), (2) and (77) yields the characteristic set:

$$C+:\quad \mathrm{d}p_x + \varrho a\,\mathrm{d}u = -F\,\mathrm{d}t\,, \qquad \mathrm{d}x/\mathrm{d}t = u + a\,, \tag{78}$$

$$C-:\quad \mathrm{d}p_x - \varrho a\,\mathrm{d}u = -F\,\mathrm{d}t\,, \qquad \mathrm{d}x/\mathrm{d}t = u - a\,, \tag{79}$$

along with eq. (77), which applies along the particle path, sometimes called the « C_0 characteristic »:

$$\mathrm{d}p_x - a^2\,\mathrm{d}\varrho = -F\,\mathrm{d}t\,, \qquad \mathrm{d}x/\mathrm{d}t = u\,. \tag{80}$$

The characteristic equations are less useful for this and other time-dependent constitutive relations than for time-independent relations because there are now no quantities which remain constant on characteristics. This means that wave transitions are no longer limited to specific curves in the (p, u) plane, as described in Sect. 3, and that type of analysis loses most of its utility. The characteristic equations can still be used in numerical analysis, though it is almost always simpler to use a von Nuemann-Richtmyer procedure.

The principal observable effect of eq. (77) on the shock wave is decay of the elastic precursor. The nature of this decay can be seen by an approximate analysis. Suppose the precursor is never of such large amplitude that its speed of propagation differs significantly from the ambient elastic speed a_0. Then the jump condition for the precursor becomes (eq. (5)):

$$p_x = \varrho_0 a_0 u$$

and when the precursor amplitude decays by $\mathrm{d}p_x$, the particle velocity behind